# Solution to UyHiP 2016

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June 3, 2016

## 1 Closed-form Of Answer

The form is :

$$ans_n = \frac{1}{2\alpha^{n+1}} + \frac{1}{2\beta^{n+1}}$$

where

$$\alpha = \sqrt{2} - 1, \beta = -\sqrt{2} - 1$$

### 1.1 proof

Let $f_{x,y}$ denote how many ways to walk to point $(x, y)$, where $x \in \mathbb{Z}_+, 1 \le y \le 3$. By symmetry, $f_{n,1} = f_{n,3}$. So

$$f_{n,1} = \begin{cases} 1 & n = 1 \\ f_{n-1,1} + f_{n-1,2} & n > 1 \end{cases}$$

$$f_{n,2} = \begin{cases} 1 & n = 1 \\ 2f_{n-1,1} + f_{n-1,2} & n > 1 \end{cases}$$

Let

$$F_1(x) = \sum_{n=0}^{\infty} f_{n,1}x^n, F_2(x) = \sum_{n=0}^{\infty} f_{n,2}x^n$$

Then

$$F_1(x) = x(F_1(x) + F_2(x) + 1)$$
$$F_2(x) = x(2F_1(x) + F_2(x) + 1)$$

So

$$F_1(x) = -\frac{x}{x^2 + 2x - 1}$$
$$F_2(x) = -\frac{x^2 + x}{x^2 + 2x - 1}$$

The answer is $f_{n+1,2}$. So let's focus on $F_2(x)$. Let $\alpha, \beta$ are two roots of $x^2 + 2x - 1 = 0$, wlog, $\alpha = \sqrt{2} - 1, \beta = -\sqrt{2} - 1$, we have

$$F_2(x) = x\left(\frac{\frac{\alpha+1}{\beta-\alpha}}{x - \alpha} + \frac{\frac{\beta+1}{\alpha-\beta}}{x - \beta}\right)$$

So

$$f_{2,n} = -\frac{\alpha+1}{\alpha^n(\beta-\alpha)} - \frac{\beta+1}{\beta^n(\alpha-\beta)}$$

That is,(because $\alpha = \sqrt{2}-1, \beta = -\sqrt{2}-1$)

$$ans_n = \frac{1}{2\alpha^{n+1}} + \frac{1}{2\beta^{n+1}}$$

## 2 7-Operations Answer

First because $|\beta| > 1$, $\frac{1}{2\beta^{n+1}}$ is small. more formally we have $|\frac{1}{2\beta^{n+1}}| < 0.5 (n \geq 1)$. So

$$ans_n = \mathrm{rnd}(\frac{1}{2\alpha^{n+1}}) = \mathrm{rnd}(\frac{1}{2} \times (\mathrm{e}^{n+1})^{-\ln\alpha})$$

Precalculate $-\ln\alpha$ and store it into the calculator. We use following operations :

`inc exp x^y MS / 2 rnd`

## 3 Answer For Bonus Question

If we have a "divide by 2" operation, we can reduce the number of operations by 1. e.g:

`inc exp x^y MS /2 rnd`

And, if we have a "$y^x$" operation(similar to "$x^y$" but swap the two operands), we can reduce 1. e.g:

`inc y^x MS /2 rnd`

(here we store $\frac{1}{\alpha}$ into memory, not $-\ln\alpha$.)